International Journal of Research in Advent Technology, Vol.2, No.3, March 2014
E-ISSN: 2321-9637

Hybrid Optimization for Multiobjective Multicast Routing

R. Baskarane [1], T. Sendhil kumar [2]
Senior Assistant Professor and HOD[1], M.Tech final year Student[2],
Department of Computer Science and Engineering[1, 2],
Christ College of Engineering & Technology[1, 2],
Pondicherry University, Pondicherry, India[1, 2].
csehodchrist@gmail.com [1], senshine21@gmail.com [2]

Abstract- The multicast routing protocol in Wireless Sensor Networks (WSN) must be energy aware since the nodes are energy constrained due to limited battery life. This gives rise to the need for efficient multicast routing protocol that is able to determine multicast routes which satisfies the quality of service guarantees and at the same time conserves energy. The transmission of real time multimedia services in wireless sensor networks requires optimal multicast routing protocol that satisfies the quality of service guarantees. The design of such protocol can be formulated as a Multiobjective Multicast Routing Problem (MMRP) that attempts to optimize the objectives simultaneously. The proposal provides a novel multiobjective Hybrid evolutionary algorithm based on Ant Colony Optimization (ACO) and particle swarm optimization (PSO) for MMRP problem. The Hybrid protocol attempts to optimize the end-to-end delay and total transmitted power simultaneously to obtain the Pareto-optimal solutions. The simulation results are very promising and show that our algorithm is able to find near optimal solution efficiently.

Index Terms- routing, Wireless Sensor Networks (WSN), Ant Colony Optimization (ACO), particle swarm optimization (PSO).

1. INTRODUCTION

Optimization is a scientific discipline that deals with the detection of optimal solutions for a problem, among alternatives. The optimality of solutions is based on one or several criteria that are usually problem and user-dependent. For example, a structural engineering problem can admit solutions that primarily adhere to fundamental engineering specifications, as well as to the aesthetic and operational expectations of the designer. Constraints can be posed by the user or the problem itself, thereby reducing the number of prospective solutions. Swarm intelligence (SI) is the collective behaviour of decentralized, self-organized systems, natural or artificial. The concept is employed in work on artificial intelligence. The expression was introduced by Gerardo Beni and Jing Wang in 1989, in the context of cellular robotic systems.SI systems consist typically of a population of simple agents interacting locally with one another and with their environment. Ant colony optimization (ACO) is a class of optimization algorithms modelled on the actions of an ant colony. ACO methods are useful in problems that need to find paths to goals. Artificial 'ants' simulation agents locate optimal solutions by moving through a parameter space representing all possible solutions. Particle swarm optimization (PSO) is a global optimization algorithm for dealing with problems in which a best solution can be represented as a point or surface in an n-dimensional space. Hypotheses are plotted in this space and seeded with an initial velocity, as well as a communication channel between the particles. The development and deployment of wireless sensor networks (WSN) have taken traditional network topologies in new directions. Many of today’s sensor applications require networking alternatives that reduce the cost and complexity while improving the overall reliability. So the hybrid evolutionary approach which initiates both ant colony followed by the particle swarm, which produces the highly scalable node transmission. The optimizer in the method, iterative algorithms such as Evolutionary Algorithms such as, Ant Colony Optimization (ACO), and Particle Swarm Optimization (PSO) are widely used to find the global optimal solutions. However, these algorithms usually take too much time to converge when a large-scale network is encountered. The Liberated output provides a feasible and QOS guaranteed with the metrics it comes with.

2. RELATED WORK

Maximum-Flow- Minimum-Cost routing algorithm is presented. The algorithm computes maximum-flow routings for all smooth unicast traffic demands within the Capacity a network subject to routing cost constraints. The edge cost can be a distance, reliability, congestion or energy metric. It is shown that every network has a finite Bandwidth-Cost capacity. The Bandwidth-Distance and the Bandwidth-Energy capacities are explored. The routing algorithm requires the formulation of two Linear Programs (LPs) [1]. The first LP finds a multicommodity Maximum-Flow, when the flows are constrained to a sub-graph of the network to enforce cost constraints. The second LP minimizes the routing cost, given that the maximum-flow is fixed. A related Constrained Multicast-Max-Flow-Min-Cost algorithm is also presented, to maximize the throughput of a multicast tree using network coding, subject to routing cost constraints [2]. These algorithms have polynomial-time solutions, whereas traditional multipath routing algorithms can be NP-Hard. The addition of routing cost constraints can significantly reduce the size of the LPs, resulting in faster solutions, with lower edge utilizations and with higher energy efficiencies. The application of these algorithms to route a, energy-efficiency and QoS guarantees is presented [3]. When the cost constraints are relaxed, no other unicast routing algorithms can achieve larger Maximum Flows, or lower costs given the Maximum-Flow rates to be supported; these unicast routing algorithms can achieve the lowest energy-costs given the Maximum-Flow rates to be supported. These routing algorithms have polynomial time solutions, in contrast to traditional multipath routing algorithms which can be NP-Hard. It is also shown that every network has a finite Bandwidth-Cost capacity which cannot be exceeded. Two capacities where explored, the Bandwidth- Distance capacity and the Bandwidth-Energy capacity. The proposed routing algorithms can achieve Maximum-Flows with minimal BD and BE costs, subject to cost constraints imposed by a network administrator. Here also present some new insights into Multicast Maximum-Flow-Minimum-Energy routing in networks using Network Coding [8]. It is shown that the energy costs of different multicast routings that support the same multicast flow rate can be significantly different. The application of these routing algorithms to route aggregated and smoothened video streams from Cloud data centers in a proposed Future-Internet network with improved throughput, energy-efficiency and QoS guarantees is presented.

Multicast is a form of group communication in which data is forwarded concurrently to a set of predefined destination. The rapid development in multimedia applications like video/audio conferencing, distance education and online gaming etc. require multicast communication with strict quality-of-service guarantee for different parameters such as bounded end-to-end delay, delay jitter and bandwidth. The underlying model of multicast routing is Steiner tree. Thus, the task of QoS based multicast routing is to find an optimal Steiner tree satisfying the QoS requirements.

Wireless Sensor Networks [7] pose additional challenges in finding optimal multicast routes due to presence of nodes which are severely energy constrained. This gives rise to the requirement of a multicast routing protocol that optimizes the QoS parameters and is also energy efficient. One approach for energy conservation [15] is to find routes in multicast communication which minimizes the total transmitted power level. Since most of the multimedia applications are delay sensitive, we consider end-to-end delay as the QoS parameter. The end-to-end delay is measured as the number of hops the data travels from the source to the destination node. Ideally we wish to have a path that minimizes the total transmitted power level and the end-to-end delay at the same time. However, these Objectives are conflicting in nature as choosing a path which has lower power level results in higher hop count and subsequently higher delay. In such circumstances, it is evident that there is a trade-off between the two conflicting objectives.

JPSO Algorithm for Efficient multicast routing problem [17] and Steiner tree problem. Starting from the source node, by selecting the next link which connects to any on-tree node until all destination nodes have been added to the tree. In the swarm of JPSOMR does not possess a velocity component. Instead, the swarm evolves based on different moves to the positions (solutions) of the particles. At every iteration of the evolution, each particle moves either based on its current position (an inertial move) or based on the position of the attractor which is chosen by using the weight vector (a cognitive, social or global move). Once the particle has jumped to a new position, a local search is applied. The particle's best path replacement operator is used to update a particle's position based on that of a chosen attractor. After each move, a local search is applied to improve the new particle's position. In the local search implemented here, a simple neighborhood operator operates upon the nodes in the tree. A neighbor of the current tree is obtained by removing a non-destination node and creating a new spanning tree of the remaining nodes using the Prim's spanning tree. For the DCLC multicast routing problems [20], the link delay function D (e) is defined within the simulator as the propagation delay of the link. We assume that queuing and transmission delays are negligible. The link cost function C (e) is defined as the current total bandwidth on the links in the computer network.

3. EXISTING SYSTEM

The transmission of real time services in Wireless Sensor Networks (WSN) requires optimal multicast routing protocol that satisfies the quality of service guarantees. The design of such protocol can be formulated as a Multiobjective Multicast Routing Problem (MMRP) that attempts to optimize the objectives simultaneously. The ant colony based multiobjective algorithm for obtaining Steiner tree that balances the total transmitted power at the terminal nodes and the hop count. The approach uses a pheromone trail matrix for each objective. Minimizing the transmission power assignment of all forwarders and the hop count of each route will result in a Steiner tree with the minimum transmission power and least number of forwarders Aggressive power assignment to conserve energy results in a Steiner tree with a higher number of forwarders and vice versa. The Existing work only with 2 objectives and it can be extended by considering more than two objective functions.

The JPSOMR algorithm swarm jump from one position to another in the discrete search space by making changes to the tree represented by the current position. This has been carried out by using path replacement operations which have been designed with regard to the specific structure and feature of the multicast network. It can be preceded with local search method for multiobjective routing.

4. PROPOSED SYSTEM

The project aims at developing hybrid evolutionary approach for multiobjective multicast routing problem, the computing burden for the micro-simulation is usually very heavy as a large number of vehicles are modelled and simulated separately. For the optimizer in the method, iterative algorithms such as Genetic Algorithms (GA), Ant Colony Optimization (ACO), and Particle Swarm Optimization (PSO) are widely used to find the global optimal solutions. To alleviate the computing burden and speed up the convergence, we build an agent-based routing simulator and employ hybrid Optimization as the optimizer. We accelerate both the simulator and the optimizer, which has been applied successfully in many areas for parallel computing.

The main objective of the MDR problem is to construct the optimal multicast tree in the distributed network that determines the best routing for the delivery of a message from the source node to multiple destination nodes while optimizing an International Journal of Hybrid Information Technology certain performance criteria and meeting all QoS requirements. Recently, with the high demand of fast and better quality of services, a number of rigid QoS criteria, such as bandwidth, delay, jitter, and packet loss rate, have been considered. This QoS multicast routing problem has drawn wide spread attention from researchers who have been using different methods to solve the problem using conventional algorithms. Many evolutionary algorithms, such as genetic algorithm, particle swarm, and ant colony optimization (ACO), have been proposed for solving the MR problem. However PS and ACO have practical limitations in real-time multicast routing. Both the efficiency of the PSO algorithm and the quality of the solution depends on procedures that are sensitive to the influence of random swarming sequence. The ACO algorithm has many parameters and cannot guarantee convergence to the global optimal. A PSO based algorithm to solve the MR problem in by means of serial path selection to realize the optimization of a multicast tree. The multicast tree can obtain a feasible solution by exchanging paths in the vector.

A. *Ant Colony Optimization*

It is a population-based meta-heuristic search technique that can be used to find approximate solutions to difficult optimization problems.

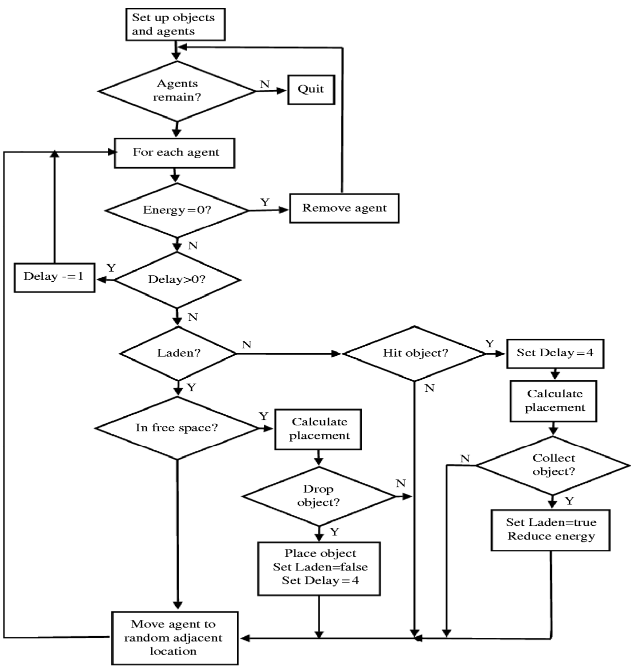

Fig. 1 Ant colony Optimization

In ACO, a set of software agents called artificial ants search for good solutions to a given optimization problem. To apply ACO, the optimization problem is transformed into the problem of finding the best path on a weighted graph. The artificial ants (hereafter ants) incrementally build solutions by moving on the graph. The solution construction process is stochastic and is biased by a pheromone model, that is, a set of parameters associated with graph components (either nodes or edges) whose values are modified at runtime by the ants. The complex social behaviors of ants have been much studied by science, and computer scientists are now finding that these behavior patterns can provide models for solving difficult combinatorial optimization problems. The attempt to develop algorithms inspired by one aspect of ant behavior, the ability to find what computer scientists would call shortest paths, has become the field of ant colony optimization (ACO), the most successful and widely recognized algorithmic technique based on ant behavior. The ant colony meta-heuristic is then introduced and viewed in the general context of combinatorial optimization. Ant Net, an ACO algorithm designed for the network routing problem, is described in detail. Ant Colony Optimization will be of interest to academic and industry researchers, graduate students, and practitioners who wish to learn how to implement ACO algorithms.

1. {Initialization}
Initialize $\tau\iota\psi$ and $\eta\iota\psi$, $\forall(\iota\psi)$.
2. {Construction}
For each ant k (currently in state ι) do
repeat
choose in probability the state to move into.
append the chosen move to the k-th ant's set tabuk.
until ant k has completed its solution.
end for
3. {Trail update}
For each ant move ($\iota\psi$) do
compute $\Delta\tau\iota\psi$
update the trail matrix.
end for
4. {Terminating condition}
If not(end test) go to step 2.

B. *Particle Swarm Optimization*

In computer science, particle swarm optimization (PSO) is a computational method that optimizes a problem by iteratively trying to improve a candidate solution with regard to a given measure of quality. PSO optimizes a problem by having a population of candidate solutions, here dubbed particles, and moving these particles around in the search-space according to simple mathematical formulae over the particle's position and velocity. Each particle's movement is influenced by its local best known position and is also guided toward the best known positions in the search-space, which are updated as better positions are found by other particles. This is expected to move the swarm toward the best solutions.

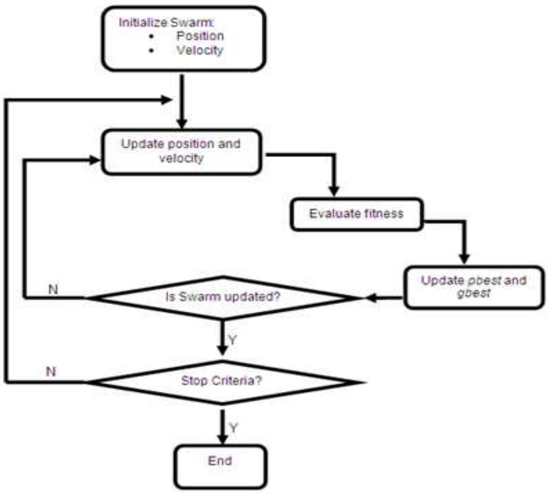

Fig. 2 Particle Swarm Optimization

Let S be the number of particles in the swarm, each having a position $\mathbf{x}_i \in \mathbb{R}^n$ in the search-space and a velocity $\mathbf{v}_i \in \mathbb{R}^n$. Let $\mathbf{p}_i$ be the best known position of particle i and let $\mathbf{g}$ be the best known position of the entire swarm. A basic PSO algorithm is then:

- For each particle $i = 1, ..., S$ do:
  - Initialize the particle's position with a uniformly distributed random vector: $\mathbf{x}_i \sim U(\mathbf{b}_{lo}, \mathbf{b}_{up})$, where $\mathbf{b}_{lo}$ and $\mathbf{b}_{up}$ are the lower and upper boundaries of the search-space.
  - Initialize the particle's best known position to its initial position: $\mathbf{p}_i \leftarrow \mathbf{x}_i$
  - If $(f(\mathbf{p}_i) < f(\mathbf{g}))$ update the swarm's best known position: $\mathbf{g} \leftarrow \mathbf{p}_i$
  - Initialize the particle's velocity: $\mathbf{v}_i \sim U(-|\mathbf{b}_{up}-\mathbf{b}_{lo}|, |\mathbf{b}_{up}-\mathbf{b}_{lo}|)$
- Until a termination criterion is met (e.g. number of iterations performed, or a solution with adequate objective function value is found), repeat:
  - For each particle $i = 1, ..., S$ do:
    - Pick random numbers: $r_p, r_g \sim U(0,1)$
    - For each dimension $d = 1, ..., n$ do:
      - Update the particle's velocity: $\mathbf{v}_{i,d} \leftarrow \omega\, \mathbf{v}_{i,d} + \varphi_p\, r_p\, (\mathbf{p}_{i,d}-\mathbf{x}_{i,d}) + \varphi_g\, r_g\, (\mathbf{g}_d-\mathbf{x}_{i,d})$
    - Update the particle's position: $\mathbf{x}_i \leftarrow \mathbf{x}_i + \mathbf{v}_i$
    - If $(f(\mathbf{x}_i) < f(\mathbf{p}_i))$ do:
      - Update the particle's best known position: $\mathbf{p}_i \leftarrow \mathbf{x}_i$
      - If $(f(\mathbf{p}_i) < f(\mathbf{g}))$ update the swarm's best known position: $\mathbf{g} \leftarrow \mathbf{p}_i$
- Now **g** holds the best found solution.
- The parameters ω, φ_p, and φ_g are selected by the practitioner and control the behaviour and efficacy of the PSO method.

C. Hybrid Evolutionary Approach

This module is a cooperative development of ACO-PSO based hybrid system, where it uses the 2 similar Evolutionary approach of Ant colony Optimization (ACO) and Particle Swarm Optimization (PSO). It works with the initiation of Ant colony system and proceed towards the Particle Swarm by each step of iteration.

The ant sub-colony can be regarded as particle, the number of ants in sub-colony equates to the number of the destination nodes m and also equates to particle's dimension, which guarantees that every ant in ant sub-colony corresponds to a destination node, every ant sub-colony can generates a multicast tree. The basic process of ACO-PSO is: the new solutions are generated by ACO, which is regarded current position of particles in PSO, in succession, the solution are regarded by velocity update and position update, the essential of regulation is that the current solution creoss with current best solution of corresponding particle and all particles respective by certain probability, which extends the search scope of solution and avoids prematurity of algorithm , The regulated solutions by PSO is used to update pheromone in ant colony network, which makes ACO-PSO more effcient.

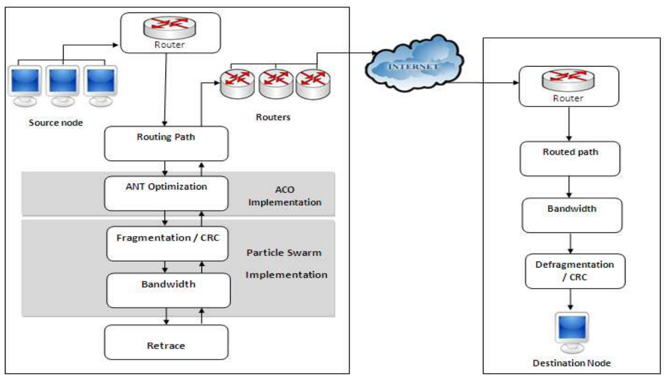

Fig. 3 System Architecture

5. IMPLEMENTATION

A. Bandwidth

Bandwidth management architecture incorporates three ideas: first, we develop a simple rule system that allows applications and the network administrator to specify how traffic generated by sensors should be treated by the sensor network. Second, we show how using multiple SAPs and SAP selection method that considers packet loss probabilities, path load, and path lengths improves the capacity of the network and the performance of individual sensor streams. Third, we show that hop by-hop flow control, rather than end-to-end congestion control, is a better way to cope with the nature of sensor network traffic and avoids unnecessary packet losses that waste valuable wireless network bandwidth. Our experimental results from a 40-node indoor wireless sensor test bed show that these three techniques are simple to implement and allow scarce network bandwidth to be used efficiently.

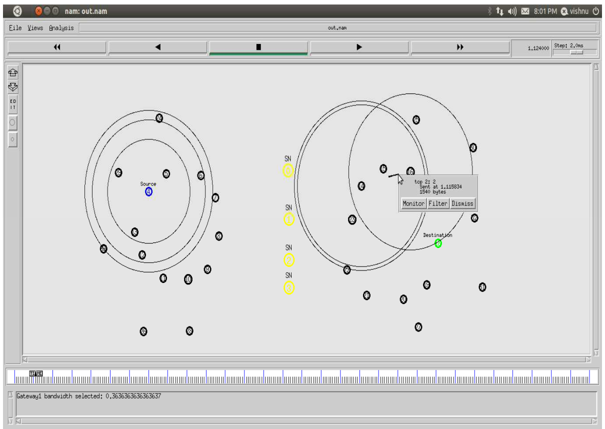

Fig. 4 Bandwidth utilization with 4 hops

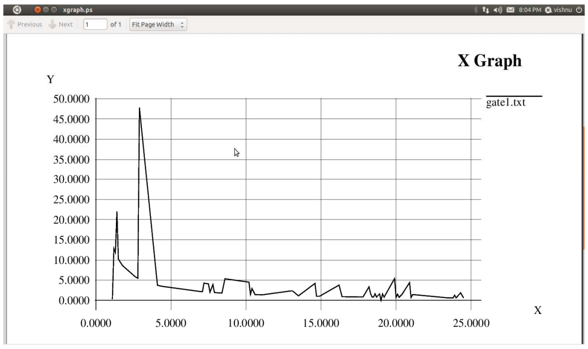

Fig. 5 Pareto graph for Bandwidth

B. *Energy*

Energy efficient layout of wireless sensor network in which sensors communicate with each other to transmit their data to a high energy communication node which acts as an interface between data processing unit and sensors. Optimization of sensor locations is essential to provide communication for a longer duration. It discusses an energy efficient layout with good coverage based on Multi-objective Particle Swarm Optimization algorithm.

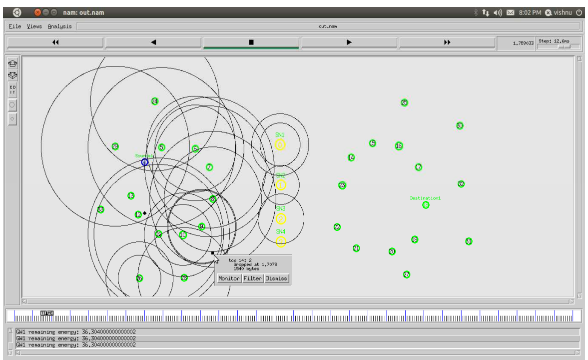

Fig. 6 Energy utilization with 4 hops

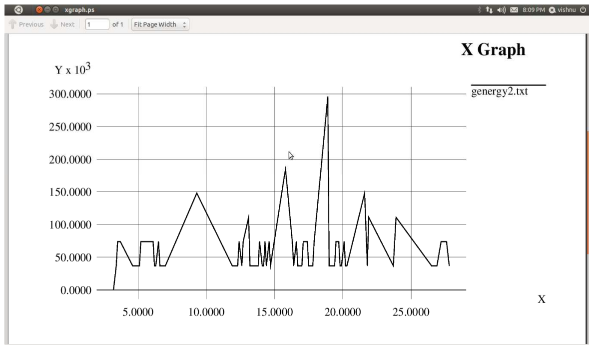

Fig. 7 Pareto graph for Energy Optimization

C. *Through put*

Allowing high-priority packets to overwrite low priority packets when buffer space runs out can lead to an improvement in high-priority streams' throughput, MAC-layer support is needed to ensure that if any high-priority packets are waiting to be sent, their nodes will win any contention competition at the MAC layer. In this section, we show how the MAC layer can be modified to accomplish exactly that. The dark grey bars show the performance of our implementation, which comes very close to the desired bandwidth allocation. The light grey bars show that without our modifications, low-priority traffic would consume some of the high-priority traffic's bandwidth.

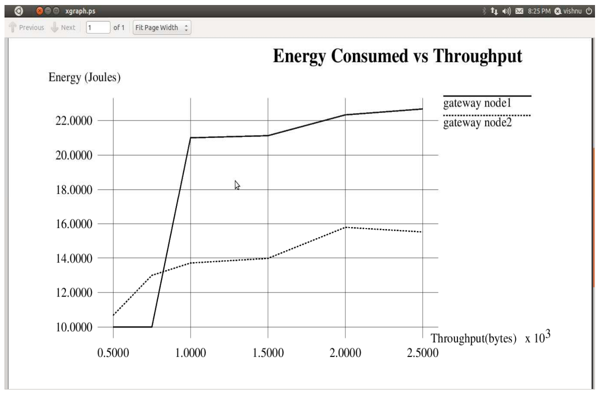

Fig. 8 Pareto-graph for Energy conserved with maximum throughput

A complete system will of course need to optimize both energy and bandwidth, and it would be interesting to study which of our techniques are problematic for energy consumption. For example, some of our techniques are optimized by nodes overhearing each other's transmissions, which requires nodes to be on and listening on the wireless channel when they might instead sleep to save energy. It might be possible to run topology formation algorithms like Span that produce and change topologies on slower time-scales, and our bandwidth management schemes on each individual instance of the network topology. We have implemented the techniques of the Hybrid Optimization toward Multiobjective multicast routing which in turn shows efficiency of 82% in Throughput and 98% of Energy Conservative performance are being carried out .

6. CONCLUSIONS

The reason of selecting these two search algorithms for empirical study lies in that, biological evolution of swarm intelligence; PSO is the effective algorithm in the field of swarm intelligence and ACO for its own advantages of stochastic search. PSO is more effective for the case of large candidate service number. An ACO with pheromone matrix and JPSO techniques have been adopted to solve the QoS multicast routing problem in communication network. The solution generated by ACO is regulated by position update strategy of PSO, which extends search scope and increase avoids local optimization efficiently. The method of positioning update in JPSO is notified in order to adapt our discrete multiobjective multicast routing problem. This proposed algorithm utilizes PSO algorithm that has emerged as a new heuristic that can efficiently solve large-scale optimization problems. This study differs from existing literature in the following aspects: First, in this study various QoS measures are considered such as cost, bandwidth, delay and jitter. The proposed model treats these constraints separately, and can be extended to add more constraints. Second, new discrete PSO operators have been presented to modify the original PSO velocity and position update rules to the discrete solution space in the multicast routing problem. Third, a new adjustable PSO-ACO hybrid multicast routing algorithm which combines PSO with genetic operators was proposed. The performance of the adjustable hybrid model is optimized by two driving parameters that give preference to either PSO or ACO The proposed hybrid algorithm can overcome the disadvantages of both PSO and ACO, and can achieve better QoS performance.

REFERENCES

[1] M.Chiang,S.Low, A.Calderbank, and J.Doyle, *Layering as optimization decomposition: a mathematical theory of network architectures*. Proceedings of the IEEE, vol.95, no.1, pp. 255–312, 2007.

[2] Y. Yi and M. Chiang, “Stochastic network utility maximisation,” European Transactions on Telecommunications, vol. 19, no. 4, pp. 421–442, 2008.

[3] L. Chen, S. Low, M. Chiangs, and J. Doyle *Cross-layer congestion control, routing and scheduling design in ad hoc wireless networks,* in Proceedings of the 25th IEEE International Conference on Computer Communications (INFOCOM ’06), pp. 1–13, Barcelona, Spain, April 2006.

[4] C. Long, B. Li, Q. Zhang, B. Zhao, B. Yang, and X. Guan *The end-to-end rate control in multiple-hop wireless networks: cross-layer formulation and optimal allocation,* IEEE Journal on Selected Areas in Communications, vol. 26, no. 4, pp. 719– 731, 2008.

[5] J. Jin, W. Wang, and M. Palaniswami, “Application-oriented flow control forwireless sensor networks,” inProceedings of the 3rd International Conference on Networking and Services (ICNS ’07), p. 71, Athens, Greece, June 2007.

[6] V. Srinivasan, C. Chiasserini, P. Nuggehalli, and R. Rao *Optimal rate allocation for energy-efficient multipath routing in wireless ad hoc networks*, IEEE Transactions on Wireless Communications, vol. 3, no. 3, pp. 891–899, 2004.

[7] J. Lee, R. Mazumdar, and N. Shroff, *Joint opportunistic power scheduling and end-to-end rate control for wireless ad hoc networks*, IEEE Transactions on Vehicular Technology, vol. 56, no. 2, pp. 801–809, 2007.

[8] J. Zhang, D. Zheng, and M. Chiang, “The impact of stochastic noisy feedback on distributed network utility maximization,” IEEE Transactions on Information Theory, vol.54, no.2, pp. 645–665, 2008.

[9] Q. Gao, J. Zhang, and S. Hanly *Cross-layer rate control in wireless networks with lossy links: leaky-pipe flow, effective network utility maximization and hop-by-hop algorithms,* IEEE Transactions on Wireless Communications,vol.8,no.6, pp. 3068–3076, 2009.

[10] M. J. Zhu, K. Hung, B. Bensaou, and F. Nait-Abdesselam, “Ratelifetime tradeofffor reliable communication in wireless sensor networks,”Computer Networks, vol. 52, no. 1, pp. 25–43, 2008

[11] F.Kelly,A.Maulloo,andD.Tan,“Ratecontrolforcommu nication networks: shadow prices, proportional fairness and stability,”Journal of the Operational Research Society, vol. 49, no. 3, pp. 237–252, 1998.

[12] M. Chiang *Balancing transport and physical layers in wireless multihop networks: jointly optimal congestion control and power control* IEEE Journal on SelectedAreas in Communications, vol. 23, no. 1, pp. 104–116, 2005.

[13] J.Zhu,S.Chen,B.Bensaou,andK.Hung, *Tradeoffbetween lifetime and rate allocation in wireless sensor networks: a cross layer approach,* in Proceedings of the 26th IEEE International Conference on Computer Communications (INFOCOM ’07), pp. 267–275, Anchorage, Alaska, USA, May 2007**.**

[14] J. Chen, S. He, Y. Sun, P. Thulasiraman, and X. (Sherman) Shen, “Optimal flow control for utility-lifetime tradeoffin wireless sensor networks,” Computer Networks, vol. 53, no. 18, pp. 3031–3041, 2009.

[15] Y. Wang, W. Wang, X. Li, and W. Song, “Interference-aware joint routing and TDMA link scheduling for static wireless networks,” IEEE Transactions on Parallel and Distributed Systems, vol. 19, no. 12, pp. 1709–1726, 2008.

[16] S. Low and D. Lapsley, *Optimization flow control—I: basic algorithm and convergence*, IEEE/ACM

Transactions on Networking, vol. 7, no. 6, pp. 861–874, 1999.

[17] L. Cai, X. (Sherman) Shen, J. Pan, and J. Mark *Performance analysis of TCP-friendly AIMD algorithms for multimedia applications*, IEEE Transactions on Multimedia, vol. 7, no. 2, pp. 339–355, 2005.

[18] L. Wang, L. Cai, X. Liu, and X. (Sherman) Shen, "Stability and TCP-friendliness and delay performance of AIMD/RED system,"Computer Networks, vol. 51, no. 15, pp. 4475–4491, 2007.

[19] J. Zhang and M. Gursoy, "Achievable rates and resource allocation strategies for imperfectly-known fading relay channels," EURASIP Journal on Wireless Conmunications and Networking, vol. 2009, Article ID 458236, 16 pages, 2009.

[20] T. Cui, L. Chen, and T. Ho *Optimization based rate control for multicast with network coding: a multipath formulation*, in Proceedings of the 46th IEEE Conference on Decision and Control (CDC '07), pp. 6041–6046, New Orleans, La, USA, December 2007.

[21] D. Bertsekas,Parallel and Distributed Computation: Numerical Methods, Athena Scientific, Belmont, Mass, USA, 2nd edition, 1999.

[22] P. Kall and S. Wallace,Stochastic Programming, John Wiley & Sons, Chichester, UK, 2003.

[23] W. Wang, M. Palaniswami, and S. Low, "Optimal flow control and routing in multi-path networks,"Performance Evaluation, vol. 52, no. 2-3, pp. 119–132, 2003.

[24] J. Chen, W. Xu, S. He, Y. Sun, P. Thulasiraman, and X. (Sherman) Shen, *Utility-based asynchronous flow control algorithm for wireless sensor networks*, IEEE Journal on Selected Areas in Communications.